



# “Yakam Matrix” as a Predictive Fractal Fragmentation Model of RSNE: Sustainable Technico-Economic Neo-reconfigurability at the Inter Housing Sector

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**Abstract:** The authors discuss a new reconfigurability of fragmentation phenomenon, technical-economic housing surfaces affecting the quality of life, likely to meet the constraints of the Inter sector of Civil Engineering, Architecture and Urbanism "CEAU" from the idea of build a residential dwelling in an urban environment on a divided surface for an adequate environment. The goal is to predict by a mathematical model of fractal fragmentation of surfaces with the natural element's method coupled with Mandelbrot geometry. Although elevation in height is a solution, the management of the space built and developed is an open question. The simulation towards Africa of 2063, using a typical case of the city province of Kinshasa, in the Democratic Republic of Congo of Sub-Saharan Africa justifies one of the trends which highlights a consistency between the projections of 2035 and reality with the use of the "Engineering Equations Solver" software. The approximated singularities obscure the admissible threshold at the limit interfaces in “Yakam Matrix”; if cities are only demographically concentrated in rural areas with a rate of 2.25-3 m<sup>2</sup> / inhabitant and one of the distances of a fragmented plot is equal to or greater than 7.5±2.5 meters according to the current trend of affected countries. In addition to the introduction and conclusion, this work has four points: (i) The "Yakam Matrix" in the habitat; (ii) The reconfigurability of the fragmentation coupling by fractal geometry and natural elements; (iii) Formulation and coupling of numerical methods and fractal geometry; (iv) predictive mathematical modeling.

**Keywords:** Modelling, Habitat, Fragmentation, Fractal, Method of Natural Elements, GCAU Inter-sector, Yakam Matrix, Simulation, Sustainability

## 1. Introduction

The phenomenon of fragmentation of living space, which seems simple, opens up a new inter-sector of the trades involved at the interfaces of the matrix. The links of the Civil

Engineering, Architecture, Urbanism and Environment Inter sectors "CEAU" must redefine the substance of the quintessence - minimal surfaces, as an adaptable principle and applicable everywhere in standards, if we claims to continue to participate in the development of the world in m<sup>2</sup> /

inhabitant on a plot of habitable land. We introduce for the first time, the coupling in the scientific understanding of this phenomenon by the fractal geometry (FG) and the natural elements method (NEM). The diversification in surfaces and the infrastructures erected in the city province of Kinshasa in the Democratic Republic of Congo (DRC), our results and proposals still raise open questions. Nevertheless, a coherence of the fragmentation of the frame with fractal responds better in the numerical approach over a length, a width, both at the perimeter and at the surface divided into natural elements; governable by the complexity of the equations of their nonlinear variations in CEAU.

In [1] the fragmentation of living space is a curious subject, not only topical but which has endured since the first reflections of Frivole 1885 [2], although "all development depends on our own development" [3]. A study for our part exhaustive on the density metric analysis and the analysis of the fragmentation of the building, with a view to better lifting a corner of the veil on the subject of the minimization of housing surfaces, particularly limited to housing in the city province from Kinshasa, Democratic Republic of Congo in Sub-Saharan Africa which remains a subject of discussion in the face of the many challenges of this 21st century, particularly global warming, population growth in cities, ending extreme poverty in lifestyle prediction in fragmented plots. What is the root cause of the fragmentation of plots if it is no longer that of extreme poverty?

Some extensive research in Africa [4, 5], in land law [6], in geography [7] in the environment [8] has not fully studied the question of modeling the fragmentation of housing plots. What is the mystery that lies in sub-Saharan Africa, where peri-urban areas are generally governed by a duality of norms (customary and state), land appropriation procedures are complex and tend to favor the formation of informal peri-urban neighborhoods? Campaigns to evict or demolish informal settlements, both in urban centers and outskirts, are the target of actions that are the subject of controversies [9], because according to the UN after COVID-19, cities will have to choose a more fair, greener and healthier. This proves that sustainable development with the parceling out of plots for adequate housing is never again an open question of life and death.

The postulate of residential urban expansion depends on a better knowledge of the morphological and functional characteristics of peri-urban spaces and on a better involvement of the State as a regulator. The technical hypothesis of surface and morphological analysis linked to population growth in French-speaking Sub-Saharan Africa is a complex problem for Africa in 2063 [10]. Because cities will have a population more than the demography of certain countries in the world. And yet the infrastructure is not keeping pace. By illustration, African capitals such as Cairo, Lagos and Kinshasa have more of the population of certain nations such as Belgium for example. Does the infrastructure keep up with demographic dynamics? What are the consequences with the fragmentation of plots already subdivided without preventing the subsequent fragmentation,

which arises randomly following the poverty line?

This massive phenomenon of fragmentation is hardly addressed, scarcely understood from a scientific point of view, too little taken into account by decision-makers and other actors of urban dynamics on the continent [11]. What are the socio-economic and environmental effects of the production of urbanization and peri-urbanization during the fragmentation of residential areas?

The threefold social, economic and environmental aspect of urban planning and development poses a problem with the sales of fragmented plots, both in the city and in the shanty town, and even in aqueous environments and along rivers.

We limit ourselves to the state of the city of Kinshasa according to the map Figure 1 which is taken into account as a pilot location in sub-Saharan Africa, under the auspices of the "Academy of Sciences & Engineering for Africa Development" ASEAD.

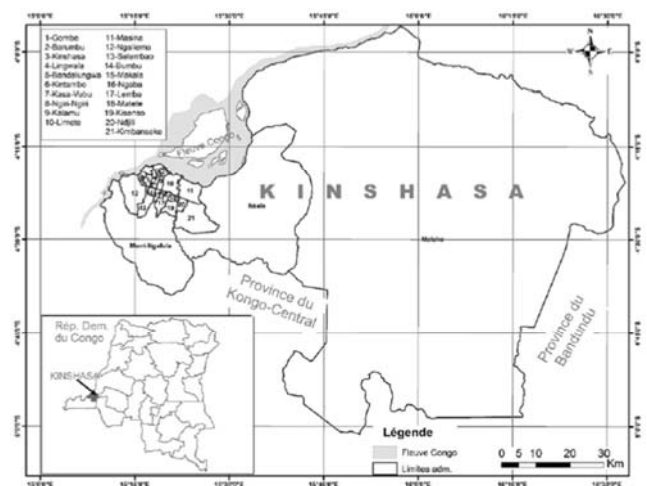


Figure 1. Map of the city of Kinshasa.

We note that the only municipality of Maluka is larger for an extension of the new city centers. The illustration of the city of Kinshasa justifies in a typical case that its spatio-temporal evolution of the morphological agglomeration of 1979 and 2015 by J. P. Messina et al. [12, 13] has never been globally approached in the years 1800 to 2020 and no projection beyond 2035 to 2063 has ever been considered. The observation of the current Kinshasa agglomeration is not only subject to a double phenomenon of extension and densification but to three times with the phenomenon of minimization of living space in the municipalities of the city of Kinshasa by statute.

This is why our original orientation in the coupling of fractal geometry and the method of natural elements leads to results which contribute towards a plausible explanatory vector of the phenomenon with a proliferation of variables.

## 2. Yakam Matrix Interfaces of Fragment

### 2.1. Interfaces

The relation (1) is a table of rows and columns. The solid

represents the ground or the living area. Rainwater and domestic water are liquids, and wind and air are essential elements for living. We assume the presence of three physical states of matter inevitably. These are: the solid (s), the liquid (l), the vapor or gas (g) and sometimes the Laser, being part of the “Yakam Matrix” introduced for the first time in 2007 [14], limited to 3 states. Currently the matrix covers all the present physical states of matter with plasma (p) and colloidal (c).

$$\mathfrak{I} = \begin{bmatrix} I_{ss} & I_{sl} & I_{sg} \\ I_{ls} & I_{ll} & I_{lg} \\ I_{gs} & I_{gl} & I_{gg} \end{bmatrix} \quad (1)$$

## 2.2. The Evolved Yakam Matrix

The matrix relation (1) has evolved by  $\mathfrak{I}_{\infty} = [I_{slgp...c}]$ , which is a matrix completed with (p) as plasma state, colloidal (c) and many other physical states of matter which will be discovered in the future by (2).

The current “Yakam Matrix” is as follows:

$$\mathfrak{I}_{\infty} = \begin{bmatrix} I_{ss} & I_{sl} & I_{sg} & I_{sp} & \cdots & I_{sc} \\ & \vdots & & & \ddots & \vdots \\ I_{cs} & I_{cl} & I_{cg} & I_{cp} & \cdots & I_{cc} \end{bmatrix} \quad (2)$$

The first line ( $I_{ss} I_{sl} I_{sg} I_{sp} \dots I_{sc}$ ), where we have solid (s), liquid (l), gas (g), plasma (p),.... And colloidal (c).

Using postdoctoral discussion, the “Yakam Matrix” seems efficient in responding to the infinite number of contributions to the many classes of knowledge.

The “Yakam Matrix” is denoted by  $I_{n \times n}(\Omega)$  the set of square matrices of order n. The n has also operating coefficients. By introducing a scalar product, the kernel and the image of a matrix  $\mathfrak{I}_{\infty}$  with subspace of which we have:

$$\ker \mathfrak{I}_{n \times n} = \{u \in H^n(\Omega), Au = 0\} \quad (3)$$

Et l’image de  $\mathfrak{I}_{n \times n} = \{Au, u \in H^n(\Omega)\}$ , parce que  $\dim \ker \mathfrak{I}_{n \times n} + \dim \mathfrak{I}_{\infty} = n$

Although the image of  $\mathfrak{I}_{n \times n} = \{Au, u \in H^n(\Omega)\}$ , with  $\dim \ker \mathfrak{I}_{n \times n} + \dim \mathfrak{I}_{\infty} = n$ , justifies the rank designation of the “Yakam Matrix”. The meaning of its core, its image, its dimension, its determinant, its transpose or its inverse comes open questions, because we limit ourselves to the classic definition of an elementary table.

## 2.3. The Landscape Matrix

Finding us on the Earth, for the man in the street believes to settle on a reassuring ground in the solid state (s). When, by chance, an earthquake or a volcanic eruption like the one in Goma in May 2021, or a flood or the newspapers highlight natural disasters, seemingly unusual. These periodic jolts do not opportunely remind us that our earth is a dead star, subject to tectonic forces, in perpetual evolution. For the most part, ignorance of these phenomena results from myopia, a lack of perspective and therefore of perspective. The Earth is said to be 4,570 million years old [15]. The landscape matrix generally designates the dominant element of a given landscape, at a given by macroscopic scale, according to the

landscape matrix in landscape ecology. The matrix can be predominantly natural in hardly artificial landscapes. In highly anthropized regions, the matrix is often made up of artificial environments or agricultural environments, more or less intensive.

The matrix is a theoretical concept, used in particular for the mapping and description of the natural elements of the landscape and the study of their interrelationships: surface, shape, spatial distribution and organization. The matrix is a construction of the mind to understand the complexity of the interrelationships between habitats (h) or ecosystems (e) at the scale of a landscape. This testifies to the existence of an interface (I) in interrelationships of the element of the matrix ( $I_{hp}$ ).

In a matrix one can distinguish the fragmentation of the mediums, like mesh in finite or natural elements. Figure 2, below shows the landscape which appears as a matrix. The landscape appears as an ocean (matrix) from which emerge islands and islets of habitat (tasks) linked to others by bridges (corridors). For species dependent on a habitat type represented in a task, the matrix can be seen as a non-habitat, as opposed to the habitats represented by the spots. This is why the basic element of a Habitat or “housing” is defined as by the geographical environment specific [5] to the life of an animal or plant species. It is a mode of organization and population by the man of the environment in which he lives. Etymologically, the word habitat “has been in use since about 1755 and derives from the Latin *habitāre*.” This is why we derive several terms such as: habitation as an Interface (I) of a family (f) and all the elements support to live (v) in a natural element ( $I_{fv}$ ). This is the home the place to sleep, because wolves have dens and some people are homeless, home, house or residence. domiciliation.

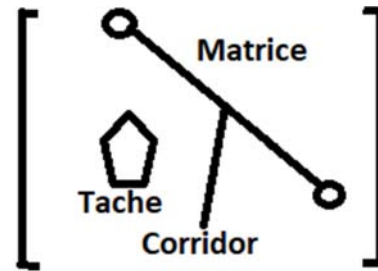


Figure 2. Landscape matrix.

The Yakam matrix is explained by its component elements of interfaces  $I_{ij}$  where I is the interface and ij are the components going from i and j and can extend more than two k, l, m,..., q. according to the combination of relation (4), which realizes the combinations of "one", two or more physical, spiritual or virtual states according to the case to be considered.

$$C_n^m = \frac{m!}{n!(m-n)!} \quad (4)$$

To meet the definition of habitat, which constitutes a simple matrix of a line or architects target the surface as a bedroom, living room, toilets as an illustration of a plot with a house of a

single person, a couple, a family with two children or more than two children, require surfaces linked to "Yakam Matrix"  $\mathfrak{S}_0$  interfaces formula (5).

$$\mathfrak{S}_0 = [I_{ss0} \ I_{sl0} \ I_{sg0} \ \dots] \quad (5)$$

Where  $I_{ss}$  the indices  $s$  is a surface that will be linked to the bed and  $l$  is the surface that will receive the tub for water which is a liquid  $l$  or the orientation of the wastewater of the habitat, even rainwater and of runoff, the index  $g$  is the air for breathing and the ventilation of the habitat or sometimes the chimney for the smoke of the kitchen. The points of suspension leave the care to the other physical states of matter, apart from those which are classically well known the three physical states of matter (solid  $s$ , liquid  $l$  and gas  $g$ ) which intervene for the life of all biodiversity, which attest to the definition of habitat that we demonstrate in the following by fractal geometry.

#### 2.4. Consensus of "Yakam Matrix" to Infinity

Faced with the concern until the death of Einstein seeking a formula capable of answering all the questions of humanity and vis-à-vis the challenges of the 21st century, the "Yakam Matrix" offers rational, liberal interfaces natural or artificial, physical or virtual of the mind that can be analyzed and sometimes contested. The illustration of one of the elements of "Yakam Matrix" is for example the link man ( $h$ ), the environment ( $e$ ) and town planning ( $u$ ) or the link is an Interface ( $I$ ) giving the component ( $I_{ehu}$ ).

There is now consensus on some of the results of discussions through national, regional and international conferences. And many users of the matrix with open questions. "Yakam Matrix" seems a way and a common thread to better understand to find an answer, because everything is related to junctions, adhesion, contact, strength and love.

### 3. Coupling Fractal Geometry and Natural Elements Method (CFGNEM) in Habitat

The parcelling out originates from the development pool is natural magnetization ( $B$ ) with extension of town or city  $rot B \neq 0$ . The concentration of the people or the agglomeration that manifests itself naturally, over time and according to the needs and activities of humans and all biodiversity. The division of plots is analyzed by fractal geometry and the natural elements method. The latter is based on a point, genesis of the names of cities or districts or avenues in the world. The first occupant or one of the initial activity can take the name of the city. Its one the central point of Voronoi diagram in MEN agreement application.

#### 3.1. Fragmentation by Fractal Geometry Habitat

All geometry begins with points  $P$ . In 1-D, 2-D or 3-D, with  $D$  as dimension. Although the methodological approach leads in topological spaces of infinite dimensions. This is the aspect that will not be addressed in this communication. We are only

recalling the segment, of which we propose in the bill that no plot of less than 5 meters is acceptable in the world for some countries.

##### 3.1.1. Unidimensional Segment in Habitat Structure

Let one of the dimensions be either the segment or  $N=3$  and  $s=3$  from Figure 3 below in fractal.

This segment cure in habitant suffers from a lack of quantitative determination in its module

Or  $|j-i| \geq 5$  meters for the case of the segment in  $s=3$ . The unit vector must be imposed greater than or equal to  $7.5 \pm 2.5$  m.



Figure 3. Segment or one of the dimensions of a parcel.

It is one of the length or width of a housing plot area.

##### 3.1.2. Square

Let us assume the surface or fragmented square configurability of Figure 4, having  $N=9$  and  $s=3$ .

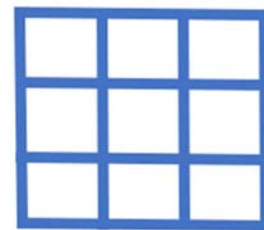


Figure 4. Fragmentation by Fractal Geometry in Fragmentation.

##### 3.1.3. Cube Volumes Structure

The Figure 5 is the self-similarity of the parcelling out. It illustrates below in the housing domain.

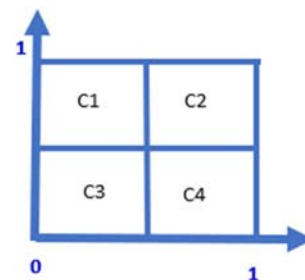


Figure 5. Similarity fragmentation parcel.

$\Omega=[0.1]$  elucidates that: the square plot

$$P_C = \omega_1 (P_C) \cup \omega_2 (P_C) \cup \omega_3 (P_C) \cup \omega_4 (P_C) \quad (6)$$

has  $[0.1] * [0.1]$

which justifies. They are called terraced houses or neighboring parcels of land with  $X$  numbering with  $A$ ,  $A$  encore or  $B1$ ,  $B$  encore, etc.

The development in fractal geometry attests to us that  $N=27$  and  $s$  always remains at 3, when we raise the building in height (RBH), according to Gervais, 2009 [16]. Figure 6 illustrates the fragmentation of a parcel in 4 fragmented  $\omega_i$  and  $i =$

1,2,3,4 plots to;

$$\begin{aligned}\omega_1(l) &= \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} l \\ l \end{pmatrix} \\ \omega_2(l) &= \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} l \\ l \end{pmatrix} + \begin{pmatrix} 1/2 \\ 0 \end{pmatrix} \\ \omega_3(l) &= \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} l \\ l \end{pmatrix} + \begin{pmatrix} 0 \\ 1/2 \end{pmatrix} \\ \omega_4(l) &= \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} l \\ l \end{pmatrix} + \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}\end{aligned}$$

This is why any fragmented plot, whatever its geometric shape, triangle, square, rectangle, trapezoid or whatever, we have  $P_C = \omega_1(P_C)$  and the index 1 which is linked as  $i=1,2,3$  and 4 self-similar divided plots in the geometry of fractalization.

All the more so the fragmentation of natural habitats is today recognized as a major threat in our understanding, not only for humans and also for biodiversity, by the convention on biological diversity which become other open questions for other specialties in biophysics or ecology.

We have understood that the fragmentation or the parceling out of the plots, which must be dilated in  $s$ , we will always have  $N = s^d$  which is written in the form of fraction dimensional (d) relation (7).

$$d = \frac{\log N}{\log s} \quad (7)$$

This is easier to justify for the width (l) or the length (L), in accordance with the segment of a dimension where  $N = s^1$ . For two-dimensional  $N=9$  and  $s=3$ ; as proof  $N = s^2$ . Finally for the three-dimensional cube when we raise a high-rise building (RHB),  $N=27$  and  $s=3$  proving that  $N = s^3$ , with all the requirements of the CEAU interface of the product. In Civil engineering for tridimensional where the mechanical behavior of  $(i, j, k) = \sum_{i=1}^3 \sum_{j=1}^3 \sum_{k=1}^3 \varepsilon_{ijk} x_i y_j z_k$ , is with  $(i, j, k) = \varepsilon_{ijk} x_i y_j z_k$  studied Hooke law in classical strength materials and applied and experimental mechanics or computational mechanics of stress and strain, with Young modulus  $E_{kh}^{ij}$  with 81 components for  $s=3$  and  $d=3$ .

“Mandelbrot [21] coined the term, Fractal to define natural forms and the hidden but simple code behind their seemingly complex appearance. Law Fractal Dimension  $D_b$  (8), the rudiment of fractals, usually described as space filling property, provides a foundation for analyzing roughness of surfaces. However, roughness exists in all dimensions. Architecture, innately related to geometry/forms, shows roughness at many scales from urban fabric to a building’s facades and plans. Several types of research have concluded that the fractal dimension of facades and plans shows levels of formal information spread in several scales. Moreover, it is argued that fractal analysis of a plan elucidates the experience of space”.

This is why the house on a fragmented parcel is a complex architectural object. It is at the same time an object of use which must meet the needs of the family group. It is a

consumer good considered as an investment, as well as an object of social and personal expression with a strong symbolic dimension. Its development is influenced by cultural values, technological innovations, political decisions and economic forces. The deterioration in both architectural and urban quality of this type of habitat gives an image of disorder. We are therefore witnessing a large-scale phenomenon. This type of habitat is created for the most part within the framework of non-homogeneous housing estates, with constructions giving the sense of an eternal incompleteness. As well as a fairly clear lack of architectural know-how, but also constructive, sometimes imposing volumes with a certain vertical elasticity, of the grounds floors which in the majority of predictive models of fractal fragmentation of residential surfaces are examined ranging of 1-D, 2-D two-dimensional.

In DR Congo the registration certificate is granted if the built area is at least more than half of the total area. In three-dimensional 3-D where we raise the house in  $R + i$ , in natural elements whose specialty of urban engineering in connection with the inter sector CEAU/GCAU would propose for our part the approach of a city by its technical networks (energy, roads, transport, water and sanitation, waste) and by its built spaces (buildings in fragmentation, equipment, public space, development), from their design to their maintenance (design, construction, management, coordination and optimized maintenance-operation) and from a triple perspective (scientific, technical and societal), with a view to a sustainable and / or intelligent city, that fragmentation should be watched as a threat. The GCAU inter-sector is confronted with random variables in terms of the evolution of the socio-economic context, concerns about innovation and sustainable development that may meet the expectations of the next day.

The more the number increases, the more the fractalization also increases in a linear or exponential demography model. The question arises of the quality of life vis-à-vis the inter-sector with the environment and health, with the case of COVID-19 for example. Let consider  $N_{s\#}$  is the number of fragmented of the initial parcel by  $\# \in N$ . The  $\frac{1}{N_{s\#}}$  is the number of parcels divided into a single initial parcel or  $\# \in N$ . The  $N_{s\#}$  justifies the number of minimum plots where  $\#$  is the basis for subdivision by classical logarithmic approach give the form relation (8);

$$D_b = \frac{\ln N_{s2} - \ln N_{s1}}{\ln(\frac{1}{N_{s2}}) - \ln(\frac{1}{N_{s1}})} \quad (8)$$

Sometimes the conflict of a plot sold by 5 people and must be shared after conflicts, if it is in a desired part of the city. The fractal formula on the log-log plot remains linear.

### 3.2. Application of the Natural Elements Method

The idea of building a house and optimizing it on a fragmented surface is strongly linked to numerical simulation in the CEAU/GCAU sector. The current requirements of dynamism and precision very often cannot be reached at the heart of software based on numerical methods to get an idea of



the resolution of the partial differential equations which model the phenomenon of habitat fragmentation. Although with the evolution of computing means and digital techniques, today allow the obtaining of satisfactory answers for an increasingly wide range of problems.

Although with the evolution of computing means and digital techniques, today allow the obtaining of satisfactory answers for an increasingly wide range of problems. Among the classical numerical methods we can cite the Finite Differences (DF) method [22], which appeared in the 1930s, and the Finite Element Method (FEM), which appeared in the 1960s [23]. These two methods have essentially the same applications. However, by its reliability, its robustness and its versatility in terms of spatial discretization, the FEM has gradually become a reference for the modeling of many classes of problems [24] either in Civil Engineering, Mechanical Engineering, in Architecture or in almost all engineering sciences, or in many other disciplines. The resolution of a problem by the FEM necessarily requires a spatial discretization based on elements. Each element - often triangular in two dimensions and tetrahedral in three dimensions - is a subdivision of the spatial domain, but which is poorly applied in the fragmentation, because all of these elements form a mesh. For FEM, numerical aspects inherent in the approach, such as shape functions, integration, precision, order of approximation, etc., are intrinsically linked to the elements of the mesh. The more the problems to be solved become more sophisticated and the FEM becomes limited. This led after 30 years of so-called meshless methods appeared with the scientific community of mechanics. meshless methods are emerging to meet the requirements of applications which do not find a satisfactory solution in classical FEM. The approximations obtained by this kind of method tend to have a higher precision than that of the FEM. In addition, the processing of displacements becomes more robust [25]. MEN [26] provides a very regular approximation, similar to that of non-meshed methods, which means that it is often referred to as a member of this family in the scientific community [27]. MEN is presented in principles related to Voronoi diagrams [28]. It is from this spatial partition that the form functions MEN [29] are constructed.

It is from this spatial partition that the form functions MEN [29] are constructed. The construction of shape functions with Voronoi diagrams [28] also called Dirichlet tiling and Thiessen polygons, the Voronoi diagram is a division of a domain from a set of points. This technique, whose origins date back to René Descartes in the 17th century, has proved useful in various disciplines, from mathematics to biology. Given a cloud of nodes  $N=\{n_1, n_2, n_3, \dots, n_N\}$  distributed in space, the Voronoi diagram is a subdivision of this domain into cells  $C_i$  associated with each node  $n_i$ . These  $C_i$  regions are defined as any point inside. Given a cloud of nodes  $N=\{n_1, n_2, n_3, \dots, n_N\}$  distributed in space, the Voronoi diagram is a subdivision of this domain into cells  $C_i$  associated with each node  $n_i$ . These regions  $C_i$  are defined such that any point inside  $C_i$  is closer to  $n_i$  than to  $n_j$ , for  $i \neq j$ . The regions  $C_i$  can

also be seen as the intersections of half-spaces delimited by the perpendicular bisectors (or mediator planes, in 3D) of the segments which connect each pair of nodes.

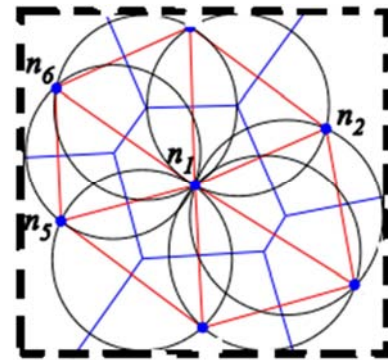
In applied mathematical to CEAU, the cells notation, called Voronoi cells, are defined as

$$C_i = \{x \in \mathbb{R}^n, d(x, n_i) < d(x, n_j), \forall j \neq i\} \quad (9)$$

Compared of parcel fragmentation in the initial conditions limits, the fragmented plot generates Voronoi cells by the simple relationship below:

$$C_i = \{x \in \mathbb{R}^n, d(x, n_i) < d(x, n_j), \forall j \neq i\} \quad (10)$$

Figure 6 shows the Voronoi diagram constructed from two-dimensional and three-dimensional node clouds. It is important to note that Voronoi cells are closed and convex for all nodes that are inside the domain. For nodes at the domain boundaries, Voronoi cells are open. The identification of the nearest neighbors of a node  $n_i$  is easily done from advanced diagram in NEM with Voronoi polyhedron.



**Figure 6.** Voronoi diagrams in blue. The Delaunay triangulation is in red and black too.

The fractal square is similar to the MEN diagram, with probability of the identified nodes going from the center  $n_1, n_2, \dots, n_5, n_6, \dots$ . Each neighboring parcel of  $x$  on  $\tilde{u}(x)$  is therefore given by  $\varphi_i(x) = A_i(x) / A(x)$ . Where  $A(x) = \sum_{j=1}^N A_j(x)$

Where  $A_i(x)$  is the area of intersection between the Voronoi cell of  $x$  and the cell corresponding to  $n_i$  in the original Voronoi diagram. And so  $A(x)$  corresponds to the area of the Voronoi cell of  $x$ , let  $A(x) = \sum_{j=1}^N A_j(x)$ . In this case, the derivatives with respect to the coordinate  $j$  of the shape function of the area or the volume of the building are given by

$$\varphi_{i,j}(x) = \frac{A_{i,j} - \varphi_i(x) A_{i,j}(x)}{A(x)} \quad (11)$$

with  $j=1,2$  associated to relation (8)  $D_b$ .

The method of natural elements or NEM (Natural Element Method) according to Chinesta et al., In 2011 [27] is halfway between the non-mesh methods and the finite element method. This method - just like the methods without mesh - proposes an interpolation defined at any point thanks to the information provided by only the surrounding nodes and is not dependent on the geometrical quality of the mesh.

NEM interpolation, like the finite element method, is nodal and is obtained by combining functions of local shapes in space. After having recalled the foundations of NEM, different extensions of the approach are exploited in the fragmentation phenomenon for the first time.

### 3.3. Formulation and Coupling of Numerical Methods and Fractal Geometry

The coupling of two methods is applied numerically by the progression of fragmented plots starting from 1, 2, 3, 4 or more. These parceled plots are built on  $R + i$  floors or without floors. Which gives us  $1^s, 2^s, 3^s, 4^s, \dots$  ou  $s=1,2,\dots$

And each fragmented parcel can in turn also be divided. Here we have to find the minimum width  $u$  to admit, knowing that

$$a(u, v) = l(v) \quad (12)$$

$$\text{where } j(u) = \frac{1}{2} l(v, v) - l(v) \quad (13)$$

with  $v$  is test function associate to relation (8) and relation (11) give the formulation of integrodifferential equation with operators constants. The fragmentation  $\Psi$  function is  $\Psi(l, L, x, x_i, y, u, u_i, v, f, A_i, B, C_i, R_j, \varphi_j, \rho, \dots, t, u(u, v, f, \dots, I_{slg}, I_{hp}, \mathfrak{I}_\infty), \dots$

$$, \frac{\partial \Psi}{\partial l}, \frac{\partial \Psi}{\partial L}, \frac{\partial \Psi}{\partial x_i}, \dots, \frac{\partial \Psi}{\partial L}, \dots, \frac{\partial^2 \Psi}{\partial l^2}, \dots, \frac{\partial^2 \Psi}{\partial t^2}) \quad (14)$$

Hence we have:  $\Delta \Psi = f$  in  $\Omega$  (15)  
for  $0 \leq A \leq \xi$  but  $u_0 = a_{11} s + b_{11}$  the series  $u_1 = a_{21} s + b_{21}$  and so  $u_n$ .

$$u_2 = a_{31} s + b_{31}$$

In a well-defined topology.

Table 1. Concept of Urban Engineering in fragmentation Habitat.

Field	Fragmentation	Properties	Behavior	(I)
Civil Engineering	Calculation	Strains	Plasticity	Iss
Architecture	Miniaturization	Philosophy of live	Adapted cost	Islg
Urban Planning	Reduced Space	Sanitation criteria	Biodiversity	Issg
Environment	Non-standard	Impacts	Compliance	Iijk
Structure	Minimization	Oversized	Hyper structure	Iss

## 4. Predictive Model

The fragmented surfaces require the architects to run high. But it has been shown the effect of the foundation which is simulated as the root of a tree. Which model can describe the deformations due to the wind illustrated by Figure 7; although the same causes produce the same effects.

The resistance force  $R_j$  from relation (19), the density  $\rho$  of the air is the density of the air,  $A$  is the surface,  $C_j$  dimensionless coefficient of the drag and  $u = dx / dt$  being the attested speed to conquer

$$R_j = \frac{1}{2} \rho C_j A \dot{x}^2 \quad (19)$$

Or  $j=1, 2$  and  $3$  according to the 3 orthonormal axes of the house to be built on a surface reduced in finished volume.

$$\|u - u_h\| \leq \inf_{v_h \in V_h} \|v - v_h\| \quad (16)$$

So for  $u \geq 5 m$ .

This distance in the Habitat  $u(x, y, z, t)$  at any point of the house built on a reduced area where  $x$  is the width,  $y$  is the length and  $z$  is the elevation in height and that the distance becomes a mathematical physical norm and can be justified by the relation (15) knowing that

$$\|u\|_{L^1(\Omega)} = (\int_1^\infty |u|^p)^{1/p} \text{ with } p \in [1, +\infty] \quad (17)$$

$$\|u\|_{L^\infty(\Omega)} = \text{Sup}_\Omega |u| \geq 5 m \quad (18)$$

This demonstrates that the Natural Elements Method corroborates fractal geometry by applying the fragmentation phenomena in the Inter disciplinary, of which Table 1 below illustrates the fragmentation actions:

Table 1: Concept of urban engineering in the division of plots.

Domains Fragmentation actions Properties Behavior.

Civil Engineering Calculation complexity Constraints Plasticity.

Architecture Miniaturization Philosophy on life adapted to Cost. Urban planning reduced space Sanitation criteria.

Environnement Non-standard Impacts Compliance.

Mechanical structure Minimization Oversized Hyper structure. This demonstrates that the Natural Elements Method corroborates fractal geometry by applying the fragmentation phenomena in the Inter sector, of which Table 1 below illustrates the fragmentation actions:

Table 1: Concept of urban engineering in the division of plots.

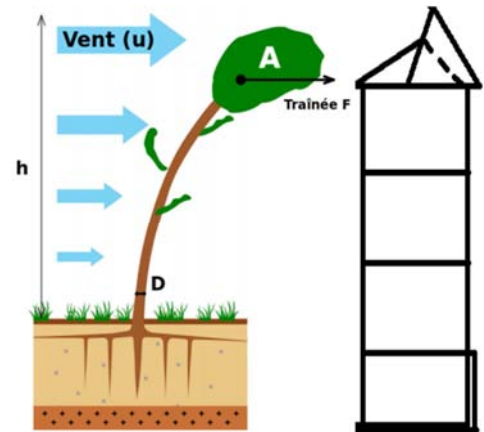


Figure 7. Approach to the wind in relation to a multi-story house with a minimum area.

#### 4.1. Current of Technico-economic Thought of Fragmentation in GCAU

Is letting houses go up high, is it a complex problem and a factor of the means? A standard requires that beyond the fixed  $R + i$ , an elevator must necessarily be associated with it for more than 4 floors. The fragmentation into cities in the process of urbanization and peri-urbanization raises formidable urban planning problems, where the actors of public policies are generally not prepared for this rapid peri-urbanization, neither technically nor economically and even less, in terms of socio-political changes [30], to impose standards in the world.

Combined with issues of urban poverty and the pace of population growth, the weakness of development policies contributes to the unorganized extensions of cities in sub-Saharan Africa [31].

Unfortunately, these unorganized growths are often inseparable from the formation of informal [8] and precarious neighborhoods according to Watson, 2009 [11]. Such evolutions are observable in Kinshasa, for example, but they are also the fact of many other Congolese [31-33] and African cities, where the outskirts in extension are affected by many socio-economic and environmental problems: absence or insufficiency infrastructure and community services [34], long trips to city centers, erosion, flooding, etc.

The concentration on fragmentation is becoming a phenomenon not only of peri-urbanization, but a massive phenomenon which is very little known from a scientific point of view and too little taken into account by decision-makers and other actors of urban dynamics on the continent [7, 9, 11, 12, 30, 32-34].

The causes and consequences of this phenomenon of fragmentation of residential plots requires finding after the simulations of a model to be validated with the case of the city of Kinshasa towards an attempt at fractals and natural elements, again approached for the first both in inter-sector "GCAU". In Kinshasa the alarm bell [33] with cry to remedy the consequences linked to the fragmentation of non-standard surfaces which is a round not yet fully addressed [34].

How does residential peri-urbanization impact urban efficiency in this city [35] without quantitatively determining the minimum distance to be accepted in the current context?

In addition to the current of economic thought in the organization of the fragmentation of urban space on multi-parameter social, environmental, scientific and technical aspects in the GCAU inter-sector, towards instruments for the internationalization of environmental externalities in this complex phenomenon, the biophysical dimension of the possible impact of building on the small surfaces of the plots means that it escapes the engineering sciences, to get an idea of all the stakeholders giving rise to the originality of the components of "Yakam Matrix" in its extension to globalizing layers. Because it must provide a universal point of view, environmental goods or services that do not recognize borders at interfaces, which no one can predict. On a plot, family or household scale, doesn't the small fragmented surface contribute to the acidification of the air,

soil or earth, water

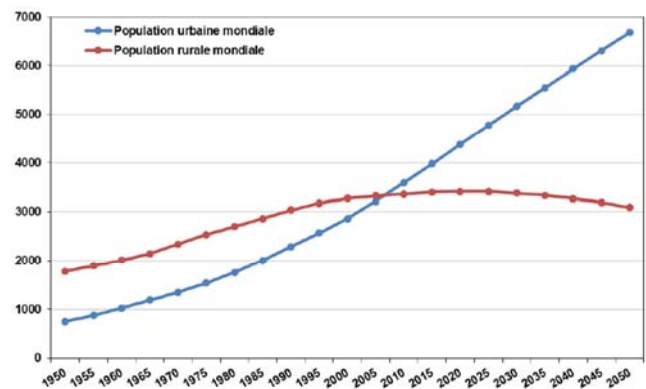


Figure 8. World urban population and world rural population.

It is remarkable from 2010, the rural exodus is one of the causes of fragmentation. There are neighborhoods with a high ethnic concentration which the MEN justify. And the fractalization is due to the degradation of the sources of income.

Figure 9 illustrates the urban population of the richest countries and the poorest countries and the rural population of developed countries and the population of developing or emerging countries. Although we play on names such as industrialized countries for the rich countries, while the poorest countries, the most indebted countries, the underdeveloped, the third world countries, the developing countries, short of the countries poor, or US President Trump called "the land of death."

#### 4.2. The African Population in the Face of Fragmentation

As we focus on Africa which abounds with the poorest countries on the planet. Figure 10 shows the urban and rural African population in simulation until 2050. A survey in Africa, that China has invaded all of Africa during the last 3 decades, in the redemption of habitable spaces, compared to European or Anglo-Saxon systems, which we leave to other specialties.

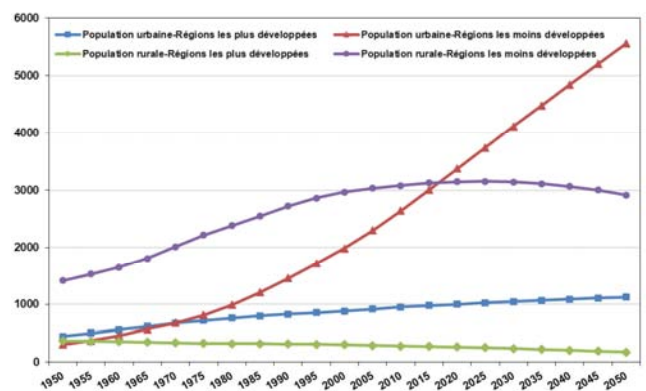


Figure 9. Urban & Rural population/surface projection.



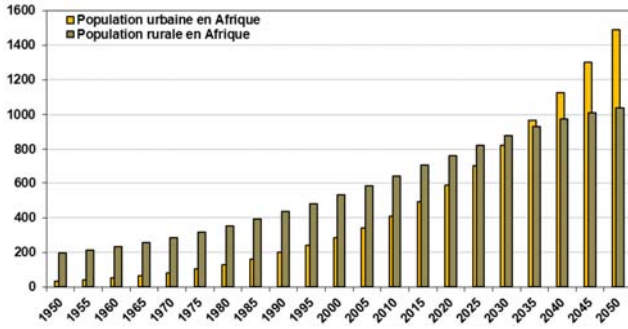


Figure 10. Urban and rural population in Africa.

#### 4.3. The Congolese Urban and Rural Population Faced with the Concentration in Towns

The population of DR C [39] in June 2021 was 80 million. Kinshasa alone covers more than 15 million inhabitants. The fragmentation is observed almost everywhere in the municipalities of the city province of Kinshasa.

Figure 11 shows the evolution of the price per square meter in the central areas.

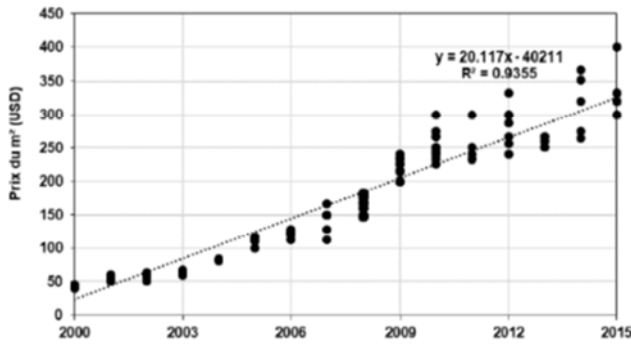


Figure 11. Evolution of the price per square meter of land in the central zone.

The price in Kinshasa ranges from US \$ 80-300 per m<sup>2</sup>. Figure 12 illustrates the development of the last decade of a survey [40].

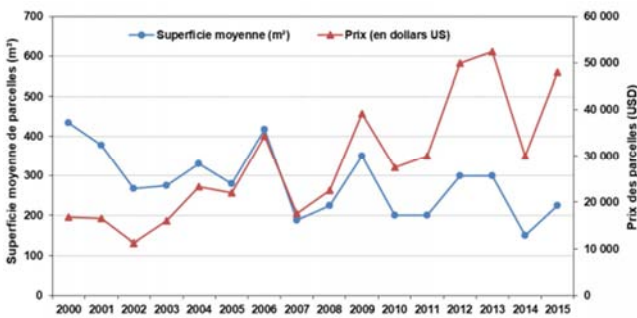


Figure 12. Evolution of the price of parcel /m<sup>2</sup> of superficies of Kinshasa Town during of 2015-2000 year [40].

Figure 13 once again represents the city of Kinshasa, which has 15 municipalities with urban status, 4 with peri-urban status and 5 with rural status. The cumulative area of peri-urban and rural areas represents more than 96% of the total area of the city, demonstrating the strong potential of the phenomenon of peri-urbanization with high concentration in

the city center and peri-urban for fragmentation. We also observe a trend of densification in all areas of the urban-rural gradient with an intermediate density situation in peri-urban areas.

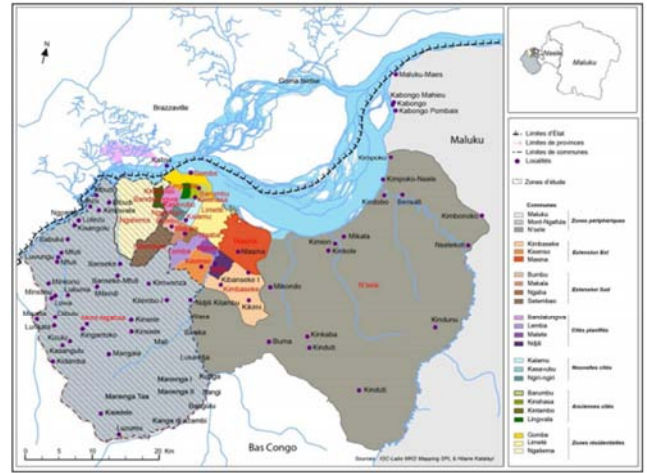


Figure 13. Distribution of municipalities in the city of Kinshasa.

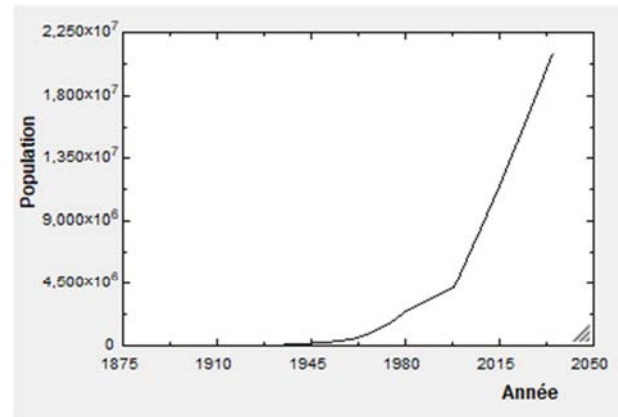


Figure 14. Population by year.

Kinshasa with its 24 municipalities, whose fragmented plots are in almost all the municipalities and there are cases of plots of 3 meters in width, in the municipality of Lemba.

While Figure 14 shows the temporal behavior of the density of number of inhabitants per ha and the last Figure 14 shows the area as a function of years.

And finally the fragmentation is put to the test by the behavior of the population in millions of inhabitants in the city of Kinshasa, which proves the minimization of living space by 2050 or 2063, by Figure 15 below. It presents 3 steps Pi evolutions:

$$P_1 = a_{11} S + b_{11} \text{ if } 0 \leq S \leq 55000, P_2 = a_{21} S + b_{22} \text{ if } 0 \leq S \leq 60000 \text{ and}$$

$$P_3 = a_{31} S + b_{32} \text{ if } 0 \leq S \leq 1000. \quad (20)$$

S is area this phenomenon is justified in some African or underdeveloped country, as the illustration of that of Algeria [43].

A phenomenon of accelerated urbanization which exceeded 60% in 2006; statistics forecast a rate of 70% in 2010 and 80% by 2025. Faced with this situation, the inter-sector GCAU is

confronted for each nation, many poor countries with the irresponsibility of the State. The people are doomed to take charge of themselves in a social character that escapes engineering, forming a sort of rather remarkable visual disorder [36]. Sometimes this type of habitat, users themselves have become "designers" out of necessity and there is a great deal of unpredictable damage in loss of human life and property.

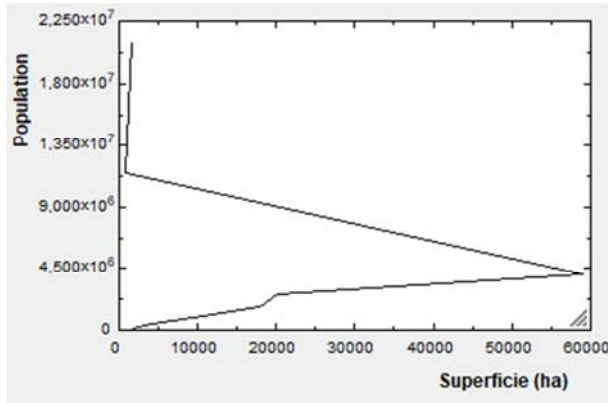


Figure 15. Population according to area.

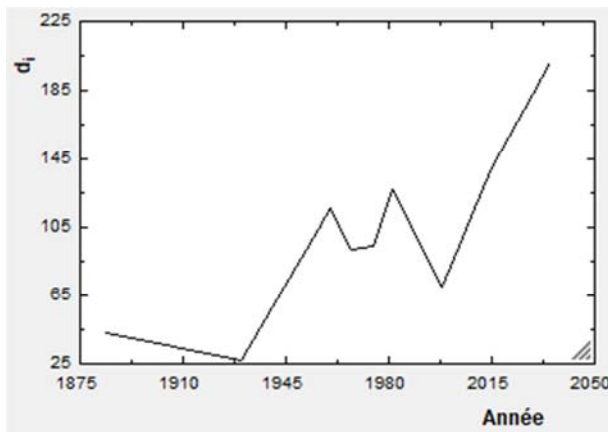


Figure 16. Temporal behavior of density.

Figure 16 and Figure 17 respectively illustrate the density over time and the area over time.

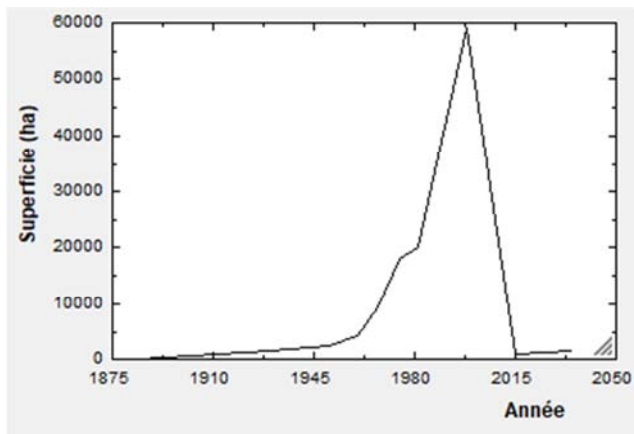


Figure 17. The area over in function the time.

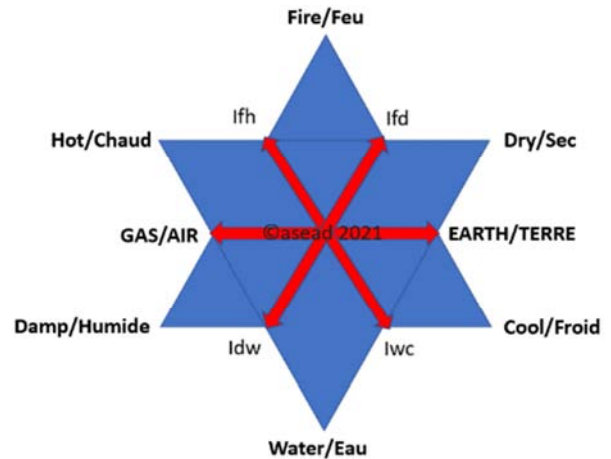


Figure 18. Seal of Salomon in CEAU.

These observations are now confirmed also in the second case exhibited in fragmentation of Habi-tat portion of Earth surface presented by the Figure 18. The air, water and fire or hot and cool produce others Interfaces in "Yakam Matrix" for open questions in Engineering and Buildings' Physics research in CEAU field. One additional remark may however be made about climate changing with fragmentation process about the temperature of hot or natural catastrophes in the future.

## 5. Conclusion

The phenomenon of dividing up plots in itself is not bad in terms of sharing. However, the process is not trivial at the interfaces of many concerns about quality of life, health, environment, leisure and ventilation space. The phenomenon of peri-urbanization in sub-Saharan Africa with the fragmentation of plots is very worrying.

The population according to the area puts to the test the phenomenon of fragmentation of housing plots or / and the phenomenon of accelerated urbanization which exceeds more than 50% in one or two decades of cities like those of the DR Congo [12, 30, 32], Algeria [43], Egypt [30].

The technical and political assumptions do not agree in developing countries with the GCAU inter-sector, with the problems of Habitat or the physics of buildings.

That one of the distances of a divided parcel not be less than 5 m. Normally the admissible limited would be 10 m, but "the wine being drawn, it must be drunk".

The city of Kinshasa, in the fragmentation of the domestic space, remains well characterized, by housing standards of 6 and 11 m<sup>2</sup>, which clearly corresponds to situations of over density. Faced with such a situation, an evolution to be supported is that of high-rise construction, which would make it possible to accommodate more inhabitants while limiting the problems of overcrowding inside housing and noise pollution and the principle of control of aeration and ventilation (PCAV).

The root causes are the poverty line and the weakness of decision-makers on housing and reception policy as well as the lack of legal texts on fragmentation. Fractal modeling

converges with fragmentation. However the method of natural elements proves the concentration in the development pool, of which we recommend the dispersion of the activity pools over all the cities. We recommend placing the offices of some ministers not only in the City Center, but in peri-urban areas, this will contribute to the fluidity of transport and activities in the cities, in traffic jams, while ensuring security and guaranteeing peace. to all and local services.

We recommend placing the offices of some ministers not only in the City Center, but in peri-urban areas, this will contribute to the fluidity of transport and activities in the cities, in traffic jams, while ensuring security and guaranteeing peace. to all and local services.

The “Yakam Matrix” remains an excellent table useful for responding to the challenges to be met by the recognition of assistance to interfaces as stakeholders, facilitating the investigation of the solution or opening track towards the better momentum with all simplicity, when of its appropriation as a public good, of material and spiritual or virtual physical independence.

The next day's population will get used to living in a small area, if the ban actions are not implemented.

The GCAU cross-stream contains many open questions in both engineering and human sciences. The new reconfigurable approach to sustainable development by the application of fractal geometry and the method of natural elements remains a new approach for the next day.

All the problems of fragmentation, of the acceleration of urbanization, of the architecture of the users themselves, of the housing estates of the disordered people "Tshasasa" meaning district without avenues and without numbers still existing in some cities of the DR Congo (Rwashi from Lubumbashi, Tshasasa from Mbujimayi the shrinking city [45] to illustrate, are nonlinear. The Natural Elements Method with Voronoi diagram which is the center of reception and the fractal geometry in 'Yakam Matrix' can necessarily linearize infinitely in spaces classical topological. However the manipulation of water and air in such a medium via Navier-Stokes, the wear of the mediums with large deformations of the contacts to the physical states of matter for the benefit of all biodiversity will not pass to linearization process, but by an iterative and no longer direct It is obvious in a dynamic framework the coupling makes use of either directly with an incrementation randomness of time in finite natural elements, facilitating the simulation for 2025, 2030, 2050 or 2063 in Africa's Agenda for its sustainable development.

The city of Kinshasa between 13-18 million inhabitants, the growth is exponential of fragmented plots, of which a barrier measure like that of COVID-19 is needed, in real and lived asymptotic behavior.

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